WYLE LABORATORIES - RESEARCH STAFF

REPORT WR 66-19

DYNAMIC RESPONSE OF BUCKLED STIFFENED PANELS TYPICAL OF S-II STAGE FORWARD SKIRT UNDER ACOUSTIC LOADS

Interim Report Submitted under
Brown Engineering Co. Work Order No. 933-50-19-9132
Technical Directive W-25
NASA Contract No. NAS 8-20073

Prepared by C. L. Amba-Rao
C.L. Amba-Rao

Approved by B. J. Spice
B.J. Spice

Approved by Kenneth McK. Eldred
Kenneth McK. Eldred
Director of Research

Date March, 1966

COPY NO. 12

SUMMARY

N67-11685

This report deals with the prediction of the response (the maximum deflection and maximum stresses) of a stiffened sheet, under compressive loads in the direction of the stiffeners - in the post buckling stage - when subjected to normal acoustic loadings.

The buckled sheet between a set of stiffeners is replaced by an equivalent uniform orthotropic plate; the differential equation of motion under an acoustic load is solved exactly, within the limitations of the small deflection theory of plates.

The work is not complete. The evaluation of the numerical results is yet to be done; however, the groundwork and a formal solution are presented.

Author

TABLE OF CONTENTS

LIST OF FIGURES

1.0 INTRODUCTION

The following note is a brief exploratory theoretical study, in continuation of some preliminary tests conducted by Wyle Laboratories under contract NAS8-20073 (References 1 and 2).

The problem under investigation is as follows:

The study investigates the effects of panel buckling due to in-plane loads, on the dynamic response of typical SII panels. The compressive loads arise from the thrust loading and vehicle bending during adjustments to the flight path. Also present, but not considered in this report, are the tension loads due to internal pressure and vehicle bending. In total, it has been estimated that many combinations of tension and compression can exist. Severe local dynamic conditions exist for the panel when an oscillating shock is present. Thus, the analysis considers the panel response to an acoustic loading.

2.0 DESCRIPTION OF SPECIMEN AND LOADING

The specimen under consideration for purposes of analysis is shown in Figure 1. The dimensions of the panel are 8.64" x 36" between the supports. The boundary conditions at the two vertical edges may be treated as fully fixed while at top and bottom edges, the edge conditions are simply supported. The stiffeners carry the compressive loads.

3.0 REVIEW OF EARLIER WORK

The critical (Euler buckling) load for a strut is its ultimate load, while the ultimate load for a plate is much greater than its critical load. Current structural design practice calls for weight limitations in space vehicle and aircraft designs. Plates in a post-buckled state are utilized in service conditions.

A literature survey of publications in fields which are closely related to the problem in hand is made; a brief list of important representative References 3, 4, and 5 are summarized. Invariably all the available technical literature utilized the non-linear partial differential equations of large deflection theory of plates, known as Von-Kármán, Tsien (Ref. 6), and approximate energy techniques. In the literature, it is repeatedly emphasized that the linear theory of plates (the deflections are small as compared to thickness) no longer applies, when the behavior of the plate above the buckling load is to be investigated. An alternate approach based on anisotropic and small deflection theory of plates yields reasonably accurate solutions.

4.0 MATHEMATICAL FORMULATION OF THE PROBLEM - BUCKLED STATE

In order to find the dynamic response of the buckled stiffened sheet, the following are alternate methods of analysis, each with its own limitations, and shortcomings.

(1) strain energy methods
(2) methods of shallow shell theory
(3) plate with initial imperfections
(4) anisotropic plate theory

For the present analysis, it is proposed to apply the anisotropic plate theory type of analysis, as it is simple; consistent with the fact that it gives reasonably accurate answers (Ref. 7) in a limited time.

For the panel under study

$$\frac{a}{b} = \frac{36}{8.64} \simeq 4.1 \qquad (1)$$

Let m be the number of half waves in x direction and n be the number of half waves in y direction. Half wave in y direction equals b; and $n = 1$ for simply supported plates.

change (Ref. 8) from m (=3) to m + 1 (=4) Half waves in x direction occurs when

$$\frac{a}{b} = \sqrt{m\,(m+1)} = \sqrt{3 \times 4} = 3.4.$$

Change from m (=4) to m + 1 (=5) half waves in x direction occurs when $\frac{a}{b} = \sqrt{4 \times 5} = 4.4.$

$\therefore$ at $\frac{a}{b} = 4.1$, 4 half waves in x direction and 1 half wave in y direction do exist.

The buckling load F_{cr} is given by

$$K\,E\left(\frac{t}{b}\right)^2$$

where K is a function of the aspect ratio, $\frac{a}{b}$ and edge conditions; t is thickness of the unsupported plate; E is elastic modulus; a is the longer side of plate; and b is the shorter side of the plate. On the basis of geometry of the panel, one can assume that there will be 1 half wave in y direction and 4 half waves in x direction, as explained earlier. This information is needed to evaluate the material constants in Section 6.3.

5.0 METHOD OF EQUIVALENT ORTHOTROPIC (ANISOTROPIC) PLATES (BUCKLED PLATES)

This method is based on the assumption that the buckled plate is equivalent to an orthotropic plate of uniform thickness. (Materials which have three mutually orthogonal planes of elastic symmetry are said to be orthotropic.) The equivalent elastic constants of the orthotropic plates should be obtained from the buckled state. The elastic constants are used in the linear non-homogeneous partial differential equation shown in equation (2). The orthotropic theory and the method of analysis are not new and are widely used in the calculation of static and dynamic responses of corrugated plates, reinforced concrete slabs, laminated structural panels, wooden plates etc.

5.1 Dynamic Equations of Motion

Let $w(x, y, t)$ be the transverse displacement of a vibrating orthotropic plate of uniform thickness h. The governing equation for the deflection w, under the combined action of the acoustic loading q and edgewise compression force N_x (neglecting the rotary inertia and shear deformation) is:

$$D_x \frac{\partial^4 w}{\partial x^4} + \left(D_1 + D_{xy}\right) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4}$$

$$- N_x \frac{\partial^2 w}{\partial x^2} = q(x, y, t) - ph \frac{\partial^2 w}{\partial t^2} \qquad (2)$$

Assume the plate is subjected to a normal acoustic loading.

Introducing the notation,

$$H = D_1 + 2 D_{xy} \qquad (3)$$

Equation (2) reduces to:

$$\left.\begin{aligned} D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} &+ D_y \frac{\partial^4 w}{\partial y^4} \\ &- N_x \frac{\partial^2 w}{\partial x^2} \end{aligned}\right\} = q - ph \frac{\partial^2 w}{\partial t^2} \qquad (4)$$

The solution of (4) with the associated boundary values and initial conditions is the response (deflection) of the plate.

5.1.1 Boundary and Initial Conditions

Referring to Figure 2, the boundary conditions at the edges $x = 0$ and, $x = a$ are assumed to be simply supported, while at $y = \pm \frac{b}{2}$, they may be assumed to be clamped, in view of the fact that the plate is connected to the stiffeners. Initial conditions for the corrugated plate may be assumed to be zero displacements and zero velocity at $t = 0$.

6.0 MATHEMATICAL ANALYSIS

Equation (4) is a linear, partial, non-homogeneous differential equation. There are various classical methods available in the literature for its solution that are consistent with the assumed boundary and initial conditions. We follow the "variables separable" method, also called the product solution.

We assume that

$$w\ (x, y, t) = X(x)\ Y(y)\ T(t) \tag{5}$$

Since two opposite edges are simply supported, using Levy's method (Ref. 7) of solution, let:

$$w\ (x, y, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \sin \frac{m\pi x}{a}\ Y_{mn}(y)\ T_{mn}(t) \tag{6}$$

Equation (6) satisfies the boundary conditions at the edges $x = 0$ and $x = a$.

6.1 Homogeneous Solution

Substituting (6) in (4) one obtains:

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \Bigg[D_x \left(\frac{m\pi}{a}\right)^4 Y_{mn}(y)\ T_{mn}(t)$$

$$- 2H \left(\frac{m\pi}{a}\right)^2 Y''_{mn}(y)\ T_{mn}(t)$$

$$+ D_y\ Y''''_{mn}\ T_{mn}(t) + \rho h\ Y(y)\ \ddot{T}_{mn}(t)$$

$$+ N_x \left(\frac{m\pi}{a}\right)^2 Y_{mn}(y)\ T_{mn}(t) \Bigg] = 0 \tag{7}$$

where prime denotes differentiation with respect to y and dot denotes differentiation with respect to t. Divide by $Y_{mn}(y)\, T_{mn}(t)$ and obtain: (attention is focused on a single harmonic; the subscripts mn are deleted from (8) to (12).)

$$D_x\left(\frac{m\pi}{a}\right)^4 + N_x\left(\frac{m\pi}{a}\right)^2 - 2H\left(\frac{m\pi}{a}\right)^2 \frac{Y''(y)}{Y(y)}$$

$$+ D_y \frac{Y''''(y)}{Y(y)} + \rho h \frac{\ddot{T}(t)}{T(t)} = 0 \qquad (8)$$

$$D_x\left(\frac{m\pi}{a}\right)^4 + N_x\left(\frac{m\pi}{a}\right)^2 - 2H\left(\frac{m\pi}{a}\right)^2 \frac{Y''}{Y} + D_y \frac{Y''''}{Y}$$

$$= - \rho h \frac{\ddot{T}}{T} = k, \text{ a constant} \qquad (9)$$

where k is real (positive, negative or zero) or complex.

$$\therefore \quad -\rho h \ddot{T} = kt \qquad (10)$$

$$\therefore \quad \ddot{T} = k_1 t \qquad (11)$$

where $$k_1 = \frac{-k}{\rho h}$$

k_1 is real (positive or negative) or complex.

The possibility that $k_1 = 0$ is ruled out as $T(t)$ should be bounded for large values of t.

Solution of (11) reduces to:

$$T = C\, e^{\sqrt{k_1}\, t} + D\, e^{-\sqrt{k_1}\, t} \qquad (12)$$

If k_1 is real and negative, $T(t)$ has bounded solutions. If k_1 is real and positive, T has unbounded solutions, and instability sets in.

For this analysis, we assume that there is no instability and the solutions are bounded (and also periodic).

$\therefore$ k_1 is negative ; let $k_2 = -k_1$ and $k_2 > 0$.

Equation (12) may be rewritten as follows:

$$T = A \cos \sqrt{k_2}\, t + B \sin \sqrt{k_2}\, t \tag{12a}$$

where A and B are constants to be evaluated from initial conditions.

Equation (10) reduces to

$$w(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \frac{m\pi x}{a} Y_{mn}(y) \left[A \cos \sqrt{k_{2_{mn}}}\, t + B \sin \sqrt{k_{2_{mn}}}\, t \right] \tag{13}$$

where Y_{mn} is defined in Section 6.2.

6.2 The Complete Solution

Let us assume that the acoustic load may be expressed as:

$$q(x,y,t) = Q_0(x,y)\, f(t) \tag{14}$$

Assume it is possible to expand $Q_0(x,y)$, as an infinite series similar to the expansion of (13). Then

$$Q_0(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin \frac{m\pi x}{a} Y_{mn}(y) \tag{15}$$

where

$$q_{mn} = \frac{\int_0^a \int_{-b/2}^{+b/2} Q_0(x,y) \sin \frac{m\pi x}{a} Y_{mn}(y)\, dx\, dy}{\int_0^a \int_{-b/2}^{+b/2} \left[\sin \frac{m\pi x}{a} Y_{mn}(y) \right]^2 dx\, dy}$$

Note $$\int_0^a \int_{-b/2}^{+b/2} \left[\sin \frac{m\pi x}{a} Y_{mn}(y)\right] \left[\sin \frac{p\pi x}{a} Y_{pq}(y)\right] dx\, dy$$

$$= 0 \qquad \begin{matrix} m \pm p \\ n \pm q \end{matrix} \tag{16}$$

as the orthogonality property between the mode shapes still holds good.

First, let us find the homogeneous solution. Substitute the assumed solution for w, in (8) and obtain:

$$\left[D_x \sum_{m=1}^{\infty} \left(\frac{m\pi}{a}\right)^4 - 2H \sum_{m=1}^{\infty} \left(\frac{m\pi}{a}\right)^2 d^2 + D_y d^4 - \rho h \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} k_{2_{mn}} + N_x \sum_{m=1}^{\infty} \left(\frac{m\pi}{a}\right)^2 \right] Y_{mn} = 0 \tag{17}$$

where $$d = \frac{\partial}{\partial y}$$

The solution of (17) is well known and can be expressed as:

$$Y_{mn}(y) = E_{mn} \cosh \alpha_{mn} y + F_{mn} \sinh \alpha_{mn} y + G_{mn} \cosh \beta_{mn} y + H_{mn} \sinh \beta_{mn} y \tag{18}$$

$$\alpha_{mn} = \left[\frac{H}{D_y}\left(\frac{m\pi}{a}\right)^2 + \left\{\left[\frac{H}{D_y}\left(\frac{m\pi}{a}\right)^2\right]^2 - \frac{s}{D_y}\right\}^{1/2}\right]^{1/2}$$

$$\beta_{mn} = \left[\frac{H}{D_y}\left(\frac{m\pi}{a}\right)^2 - \left\{\left[\frac{H}{D_y}\left(\frac{m\pi}{a}\right)^2\right]^2 - \frac{s}{D_y}\right\}^{1/2}\right]^{1/2}$$

and

$$s = D_x\left(\frac{m\pi}{a}\right)^4 + N_x\left(\frac{m\pi}{a}\right)^2 - \rho h\, k_{2_{mn}}$$

Also note that for Levy type of solution, the following relationship holds good.

$$k_{2_{mn}} = \frac{1}{\sqrt{ph}} \left[D_y \, \alpha^4_{mn} - 2H \frac{m^2 \pi^2}{a^2} \alpha^2_{mn} + D_x \frac{m^4 \pi^4}{a^4} + N_x \frac{m^2 \pi^2}{a^2} \right]^{1/2} \tag{19}$$

It is interesting to note that the nature of α_{mn} and β_{mn} (real positive, real negative, zero, or complex) depends upon the relative magnitudes of the various stiffness factors.

Since the acoustic load is a time varying force of an arbitrary nature $f(t)$, the time dependent part of $w(x,y,t)$ should include an additional contribution of Duhamel integral type of representation, defined by

$$\frac{q_{mn}}{\sqrt{k_{2_{mn}}}} \int_0^t f(\tau) \sin \omega (t-\tau) \, d\tau \tag{20}$$

where q_{mn} is defined by (16) (without loss of generality, let $Q = 1$).

$$\therefore \quad w(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \frac{m\pi x}{a} \, Y_{mn}(y) \left[A \cos \sqrt{k_{2_{mn}}} \, t \right.$$

$$\left. + \; B \sin \sqrt{k_{2_{mn}}} \, t + \frac{q_{mn}}{\sqrt{k_{2_{mn}}}} \int_0^t f(\tau) \sin \omega (t-\tau) \, d\tau \right] \tag{21}$$

It is anticipated that Equation (21) is a rapidly converging double infinite series, and reasonably good engineering answers can be obtained by taking a finite number of terms (truncated series).

Having obtained $w(x,y,t)$, following Timoshenko's notation, the following generalized forces can be derived.

$$M_x = - D_x \frac{\partial^2 w}{\partial x^2} - D_1 \frac{\partial^2 w}{\partial y^2}$$

$$M_y = - D_1 \frac{\partial^2 w}{\partial x^2} - D_y \frac{\partial^2 w}{\partial y^2}$$

$$M_{xy} = - 2D_{xy} \frac{\partial^2 w}{\partial x \partial y} \qquad (22)$$

Now the bending stresses can be expressed in terms of the bending moments as follows:

$$\sigma_x = \frac{12 \; M_x \; z}{h^3} \qquad \tau_{xy} \frac{12 M_{xy} \; z}{(1-\nu^2) \; h^3} \qquad (23)$$

$$\sigma_y = \frac{12 \, M_y \; z}{h^3}$$

The maximum bending stresses occur at $z = \pm \frac{h}{2}$

From the stresses defined by (23) one can derive the principal stresses by using Mohr's circle or otherwise.

6.3 <u>Evaluation of Elastic Constants of Equivalent Uniform Orthotropic Plates.</u>

For ease of calculations and analysis, the buckled plate is replaced by an orthotropic plate of uniform thickness. If the plate has simply supported ends and sides, the number of half waves into which the plate buckles for

$$\frac{a}{b} = 4.19 \text{ is } 4 \text{ (or 5)},$$

while for a plate with clamped sides and simply supported ends, it is 6 (or 7). These values are obtained from the extrapolated curves of Figure 14.25 p. 372 of Reference [8]. It may be noted that the elastic restraints at both the sides, though nearer to a clamped condition, are not exactly clamped.

The next step is to evaluate the elastic constants of the equivalent (uniformly thick) orthotropic plate. The analysis closely follows that of Seydel (Ref.. 9) (Also quoted by Timoshenko on p. 367 of Reference [7].)

The material constants are as follows: (Material 7075-T 6 aluminum alloy)

$$E = 10.5 \times 10^6 \text{ lb/sq.in.}$$
$$\nu = 0.33$$
$$h = 0.040''$$

Assuming $f = 0.08''$ (see Figure 2), and applying the analysis of Seydei (Ref. 9).

$$D_x \simeq 63 \quad \text{lb. in.} \quad (S \simeq \ell = 6'')$$
$$H \simeq 42 \quad \text{lb.in.}$$
$$D_y \simeq 63 \quad \text{lb.in} \quad (S \simeq \ell = 8.6'')$$
$$D_1 \simeq 0 \quad \text{lb.in}$$

The whole analysis is centered round an exact value for f. (The author is as yet unable to obtain this value).

7.0 CONCLUSIONS AND RECOMMENDATIONS

This analysis also determines whether the small deflection theory of plates, coupled with anisotropic theory can correctly predict the results derivable by the large deflection theory. The results obtained from the analysis should be verified against experiments and a reasonable coincidence puts the "Method of Analysis" on a solid foundation. As far as the author is aware, a method similar to the one presented here, has never been applied to the problem of the dynamic response of a buckled sheet in axial compression and in acoustic environment.

REFERENCES

1. Wyle Laboratories - Research Staff - TM 66-7
2. Wyle Laboratories - Research Staff - TM 66-8
3. Chang, Y.W., "Vibrations and Stability of Buckled Rectangular Plates", University Microfilms, Inc., Ann Arbor, (1964).
4. Van Der Neut, A., "Post Buckling Behaviour of Structures", Agard Rep. 60, (1956).
5. Herrmann, G. and Armenakas, A.E., "Vibrations and Stability of Plates under Initial Stress", Proc. ASCE, Jr. Eng Mech. Division, Vol. 86, No. EM 3, pp 65-94, (1960).
6. Von Karman, Th., "Encyclopädie der Mathematischen Wissenschaften, Vol. IV, p. 349, (1910).
7. Timoshenko, S. and Woinowsky-Krieger, S., "Theory of Plates and Shells, Fourth Edition, McGraw-Hill, New York, p 364, p 367, p 371, (1959).
8. Peery, D.J., "Aircraft Structures", McGraw-Hill, New York, p 370, (1949).
9. Seydel, E., "Ber. deut. Versuchsanstalt Luftfahrt, (1931).

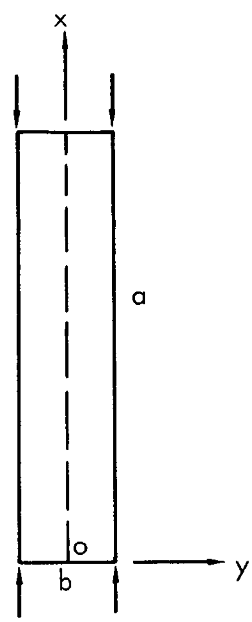

Figure 1. A Stiffened Plate in Compression - A Repeating Section

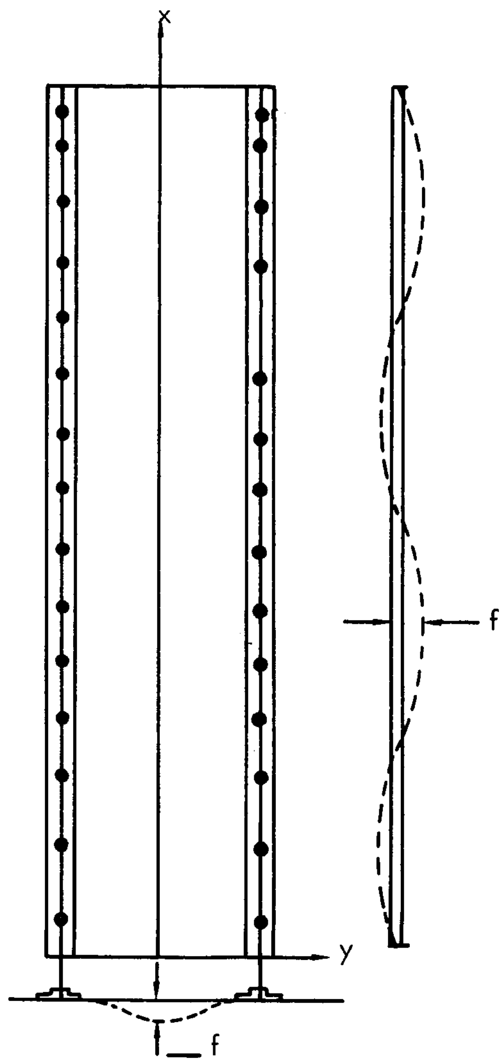

Figure 2. The Buckled Sheet is Replaced by a Corrugated Sheet for Ease of Theoretical Analysis

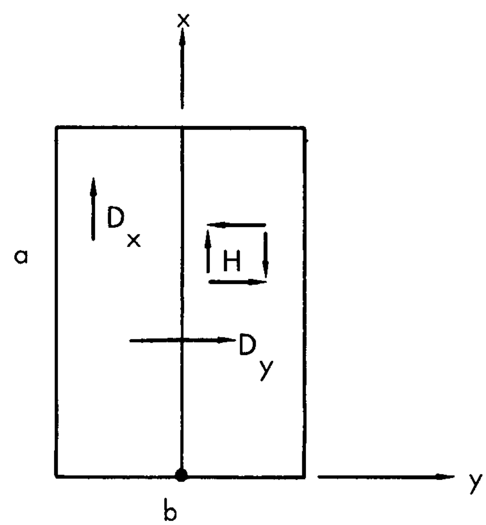

Figure 3. The Corrugated Panel is Equivalent to a Panel of Uniform Thickness Made of an Anisotropic Material